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# State estimation for a fish population via a nonlinear observer <sup>1</sup>

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## Abstract

This paper focuses on the construction of an exponential observer for an age-structured model of an exploited fish population in order to get an estimation of the number of fishes by age class. The considered model is nonlinear and involves a stock-recruitment function which is not well known. The observer that we construct is independent of this function.

**Key words:** discrete-time systems, estimation, nonlinear control systems, observer, population dynamics.

## 1 Introduction

We consider a population of exploited fish which is structured in  $n$  age classes ( $n \geq 2$ ); under some assumptions on the population, we can represent the dynamics of the population by the following system of difference equations:

$$\begin{cases} x_1(t+1) &= f(\sum_{i=1}^n b_i x_i(t)) \\ x_2(t+1) &= x_1(t) \exp(-M_1 - q_1 E(t)) \\ &\vdots \\ x_n(t+1) &= x_{n-1}(t) \exp(-M_{n-1} - q_{n-1} E(t)) \end{cases} \quad (1)$$

where  $f: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is the stock-recruitment function. It is a continuous map such that  $f(0) = 0$  and

- $b_i$  is the number of individuals produced by individuals of the  $i^{\text{th}}$  age class;
- $M_i$  is the natural mortality of the individuals of the  $i^{\text{th}}$  age class;
- $q_i$  is the catchability of the individuals of the  $i^{\text{th}}$  age class;

- $E(t)$  is the fishing effort at time  $t$  and is regarded as an input.

Several authors have proposed different kind of functions  $f$  (see [1, 4, 6, 7]). The most quoted mathematical expressions of the nonlinear recruitment function  $f$  are ( $\beta$  being a positive parameter):

$$\begin{aligned} \text{Beverton and Holt} \quad & f(x) = \frac{x}{1 + \beta x} ; \\ \text{Ricker} \quad & f(x) = e^{-\beta x} ; \\ \text{Powerfunction} \quad & f(x) = x^{-\beta} ; \\ \text{Shepherd} \quad & f(x) = \frac{x}{1 + \beta x^c} , \quad (c > 0). \end{aligned}$$

Our aim is to construct an observer, that is to say, an auxiliary system of difference equations whose state  $z(t)$  gives an estimate of the state  $x(t)$  of system (1). More precisely we shall have  $\lim_{t \rightarrow +\infty} (z(t) - x(t)) = 0$  with an exponential rate of convergence, i.e., there exists  $\alpha < 1$  such that, for all  $t \in \mathbb{N}$  and for all initial conditions  $(x(0), z(0))$ , one has

$$|z(t) - x(t)| \leq \alpha^t |z(0) - x(0)|.$$

We wish to do this job without using a precise expression of the function  $f$ : we shall use only some minimal assumption on this function. One can notice that although the theory of observer design for linear systems is a well developed field, its analogous part for nonlinear systems is an intriguing subject, and is still receiving considerable attention by many researchers. The design of state observers for discrete-time systems has been studied in many articles. One can cite, for example, [2, 3] where the problem has been addressed for general systems but the given observers are only local and here we are interested in the construction of a global observer. In [5], the observer design is done in the context of solving simultaneous nonlinear equations by using Newton's algorithm. Here we give a new procedure that simplifies the observer design for the considered system.

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## 2 Design of an exponential observer

System (1) can be rewritten with standard control notations:

$$\begin{cases} x(t+1) &= F(x(t), u(t)) ; \\ y(t) &= h(x(t), u(t)) ; \end{cases} \quad (2)$$

where  $x(t) \in \Omega = \mathbb{R}_+^n$ , is the state of the system,  $u(t) = E(t) \in \mathbb{R}_+$ , is the control (here it is the fishing effort) and  $y(t) \in \mathbb{R}_+$ , is the measurable output of the system.

We assume that we can measure the quantity of caught fishes which is expressed by :

$$y(t) = \sum_{i=1}^{n-1} x_i(t) e^{-M_i} (1 - \exp(-q_i E(t))).$$

We suppose that the fishing effort is subject to the constraints:

$$0 < E_m \leq E(t) \leq E_M$$

(this assumption is reasonable because if we don't catch any fish, we can't get any information on the population).

Below, we expose a system that we claim to be an observer for (1), for simplicity of exposition, the construction is made in the 3-dimensional case (3 age classes).

$$\begin{cases} z_1(t+1) = \frac{y(t+1) - (e^{-M_2} - v_2(t+1))v_1(t)z_1(t)}{e^{-M_1} - v_1(t+1)} \\ z_2(t+1) = v_1(t)z_1(t) \\ z_3(t+1) = v_2(t)z_2(t) \end{cases} \quad (3)$$

where we put  $v_i(t) = \exp(-M_i - q_i E(t))$ .

**Proposition 2.1** Assume that  $E_m \geq \ln 2/q_1$ , then system (3) is an observer for system (1).

**Proof.** Letting  $e(t) = x(t) - z(t)$ , we have:

$$\begin{cases} e_1(t+1) &= x_1(t+1) - z_1(t+1) \\ &= -\frac{e^{-M_2} - v_2(t+1)}{e^{-M_1} - v_1(t+1)} v_1(t) e_1(t) \\ e_2(t+1) &= v_1(t) e_1(t) \\ e_3(t+1) &= v_2(t) e_2(t) \end{cases} \quad (4)$$

thus:

$$\begin{aligned} |e_1(t+1)| &= \frac{1 - e^{-q_2 E(t+1)}}{1 - e^{-q_1 E(t+1)}} v_1(t) |e_1(t)| \\ &\leq \frac{1 - e^{-q_2 E_M}}{1 - e^{-q_1 E_m}} e^{-M_2} e^{-M_1 - q_1 E_m} |e_1(t)| \end{aligned}$$

now since  $E_m \geq \ln 2/q_1$ ,  $e^{-q_1 E_m}/(1 - e^{-q_1 E_m}) \leq 1$  which implies

$$|e_1(t+1)| \leq \alpha |e_1(t)|$$

where  $\alpha = e^{-M_2}(1 - e^{-q_2 E_M}) < 1$ . This proves the exponential convergence of  $e_1(t)$  to zero and from (4), it is clear that the same is true for  $e_2(t)$  and  $e_3(t)$ .

**Remark.** There is an alternative proof which uses the following candidate Lyapunov function  $V(e) = e^T P e$ , with:

$$P = \begin{pmatrix} \frac{3}{1-\alpha^2} & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Where,  $\alpha = e^{-M_2}(1 - e^{-q_2 E_M}) < 1$ .

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